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New model for single spherical particle settling velocity in power law (visco-inelastic) fluids

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Abstract

Particle settling in a non-Newtonian power law fluid is of interest to many industrial applications, including chemical, food, pharmaceutical, and petroleum industry. Conventionally, the Newtonian model for the drag coefficient prediction is extended to non-Newtonian fluids. The approach of merely replacing a viscosity term in Newtonian correlation with a power law apparent viscosity is reported to be inadequate.

In this investigation, the inadequacy of the Newtonian model to correlate the data of single solid spherical particle moving in power law liquids is demonstrated. An approach presented earlier by Shah has been adopted to re-analyze the previously published data of particle settling in various non-Newtonian fluids from five different investigations. The particle settling velocity data have been correlated with two dimensionless quantities – drag coefficient C_d and particle Reynolds number Re – as $\sqrt{C_d^{2-n}Re^2}$ versus Re, rather than the conventional correlation of C_d versus Re. A new model to predict the settling velocity of a spherical particle moving in inelastic power law liquids is presented, which reduces to the expected Newtonian fluid limit. It is shown that the Shah's method predicts the particle settling velocity data much closer to the experimental data than the Newtonian standard drag curve that has been widely used by many researchers. The new model is valid for a wide range of power law flow behavior index n (0.281–1.0) and particle Reynolds number Re (0.001–1000). The paper is concluded by presenting an illustrative example to calculate the settling velocity of a spherical particle in non-Newtonian liquid.

Keywords: Particle settling velocity; Sedimentation; Hydraulic fracturing; Drilling cuttings transport; Power law fluids; Non-Newtonian fluids

1. Introduction

Reliable knowledge of the free settling velocity of spherical particles in fluids is required while performing process design calculations in a range of industrial settings. Typical examples include the design of slurry

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pipelines, of liquid–solid separation equipment, of fluidized bed reactors, falling ball viscometers, etc. Additional examples are found in petroleum engineering applications. During hydraulically fracturing a hydrocarbon-bearing formation, slurry containing solid particles (more commonly known as "proppants") in a non-Newtonian carrier fluid is pumped to keep the created fracture open upon cessation of pumping. The knowledge of particle settling in slurry under static conditions is very essential in the prediction of fracture closure and the final fracture conductivity. The value of the fracture conductivity is very important to the productivity of the fractured well. While adequate information is now available which permits the calculation of the settling velocity of spherical particles in Newtonian fluids (e.g., see Clift et al., 1978; Chhabra, 2006), in some of the afore-mentioned applications (especially in petroleum engineering sector, mine tailing disposal using slurry pipe lines), the liquid exhibits shear-thinning characteristics which is frequently modelled using the usual two-parameter power law fluid model. Admittedly, a sizeable body of knowledge has accrued on the free falling velocity of spherical particles in power law fluids (Chhabra, 1986, 1990, 2006; Dhole et al., 2006), not only the body of information is nowhere near as extensive as that for Newtonian fluids but also it is much less coherent. Consequently, a simple, reliable and widely tested method is not yet available which can be used with confidence over the entire range of conditions. This work sets out to meet this objective. In particular, the purpose of this work is to demonstrate the inadequacy of the generalization of the Newtonian drag curve for the power law fluids using the notion of an apparent viscosity and also to provide a new model for the prediction of spherical particle settling velocity moving in a power law fluid. It is, however, instructive to begin with the terse description of the pertinent studies.

2. Review of previous work

Owing to their wide occurrence in chemical processes and also in petroleum processes, significant work has been reported on the hydrodynamic behavior by numerous authors to understand the mechanism of particles settling in non-Newtonian fluids. The work reported in the chemical industry has been publicized widely while the work reported in the petroleum industry has not been widely known and no attempt has been made to consolidate these results accepted by other industries. The chemical industry literature on particle settling velocity in non-Newtonian fluids prior to 1990 has been critically reviewed and reported by Chhabra (1986, 1990, 2006).

Dallon (1967) presented an empirical correlation relating the drag coefficient of spheres falling at terminal velocity and non-Newtonian Reynolds number. He worked with hydroxyethyl cellulose (HEC), carboxymethyl cellulose (CMC), and polyethylene oxide (PEO) fluids and fitted the rheological data of these fluids with Ellis model fluid. The developed equation has two constants and the terminal velocity calculations require a trial-and-error procedure. Chhabra (1990) refitted Dallon's data using the power law fluid model covering the flow behavior index, n ranging from 0.64 to 0.94 and drag coefficient between 0.46 and 3000.

The empirical correlation between the drag coefficient and particle Reynolds number developed by Prakash (1983) from the data of motion of spheres through various CMC solutions shows an additional dependence on the power law flow behavior index. It was also noted that the data could have been influenced by the confining walls but no wall correction was applied. Peden and Luo (1987) have also reported experimental data of spheres falling in aqueous solutions of CMC and HEC. The two constants in the relationship between drag coefficient and particle Reynolds number were reported to be functions of the power law flow behavior index but the dependence was found to be irregular. Also, their expression does not reduce to the expected limiting behavior for Newtonian fluids. Koziol and Glowacki (1988), Reynolds and Jones (1989) and Machac et al. (1995), have also reported similar results and correlations. But unfortunately none of these correlations have been tested using independent experimental data. More recently, Renaud et al. (2004) have revisited this problem and reported improved predictions for the data reported in chemical engineering literature. Admittedly, their method does yield reasonable predictions, but not only it is complex in form but also requires an iterative solution for the terminal fall velocity.

Yet another more recent study by Kelessidis (2004a) a argues that his proposed equation for predicting the terminal settling velocity of solid spheres in non-Newtonian shear-thinning fluids is in accordance with the drag curve of Newtonian fluids. Kelessidis and Mpandelis (2004) have also proposed a five-parameter implicit model to predict the settling velocity of single particle in pseudoplastic liquids. Their equation is similar to the one proposed by Heider and Levenspiel (1989) for Newtonian liquids using nonlinear regression and has five constants. The predictions from the proposed model are compared with their own limited non-Newtonian fluid data and with only one non-Newtonian fluid type, i.e. CMC. Data for the power law constant, n, below 0.74 are non-existent and 80% of the data fall within 30% deviation. Subsequently, Kelessidis (2004b) reported an explicit equation to predict the terminal velocity of solid spheres falling through pseudoplastic liquids. The proposed model was, however, once again tested with very limited data (55 for non-Newtonian fluids) and for $n > 0.56$ only.

All of the above mentioned studies have asserted that the drag coefficient exhibits an additional dependence on the power law flow behavior index. This dependence was in addition to that was accounted for in the definition of modified particle Reynolds number. The study of Lali et al. (1989), however, disputed these findings and argued that their data of drag coefficients of spheres moving through power law fluids correlated well with the Newtonian standard drag curve. In the midst of conflicting conclusions, Chhabra (1990) gathered a large body of experimental data available in the literature and reexamined them in greater detail to explore the possibility of using the Newtonian standard drag curve for power law fluids. Specifically, he considered the data of Dallon (1967), Prakash (1983), Machac et al. (1987), Lali et al. (1989) and his own gathered data (Chhabra, 1980). He found that the Newtonian standard drag curve provides a satisfactory representation of the drag coefficient – Reynolds number data for power law fluids, provided a Reynolds number based on the power law model is used. His study covered the fluids with the power law index values between 0.535 and 1.00 and Reynolds number range of 1–1000. While a mean error was reported to be 30%, the maximum errors up to 70% were also encountered.

In his investigation, unfortunately Chhabra (1990) did not acknowledge and include the relevant work reported in the petroleum engineering literature, except the study of Peden and Luo (1987). Novotny (1977) studied particle transport using vertical fractures. He argued that the shear rate in the fracture consists of two components: a horizontal component related to fluid motion and a vertical component related to particle settling. Hence, he presented an equation that only required knowledge of shear rate, which can be easily determined in the laboratory to predict the settling velocity.

Subsequently, Hannah and Harrington (1981) demonstrated that it was not possible to predict the settling velocity simply from knowledge of the shear rate.

Shah (1982, 1986) presented a different approach to analyze the particle settling velocity data in non-Newtonian fluids. All previous authors have employed the usual coordinates, i.e. the drag coefficient, C_d versus particle Reynolds number Re. This approach works well for Newtonian fluids, but it tends to obscure the effect of power law flow behavior index, n. This conjecture is well borne out by the data of Shah (1982, 1986) which scattered around the Newtonian drag curve without any discernable trend and also with significant deviations. However, the same data when plotted as C_d^{2-n} versus Re (to include the dependence of n on the drag curve) showed a family of curves as a function of n. His work clearly reflects the drag coefficient dependency on n and Re and also it reduces to a Newtonian drag curve, C_d versus Re, when $n = 1$. Shah also plotted his settling velocity data as $\sqrt{C_d^{2-n} Re^2}$ versus Re to avoid the trial-and-error procedure associated with the particle settling velocity calculation and proposed correlations for predicting settling velocity of particle in power law fluids. His correlations are valid for n values in the range of 0.281–1.00 and Re values in the range of 0.01–100.

The above discussed contention is examined here in more detail by adopting a large body of data available from various investigators. The particle settling velocity data of Dallon, Prakash, Lali et al., Chhabra, and Shah have been re-analyzed using the approach proposed by Shah and the results are compared with the predictions from the Newtonian drag curve.

3. Particle settling velocity data analysis

3.1. Chhabra's approach

As mentioned earlier, Chhabra (1990) analyzed the particle settling velocity data of Dallon (1967), Prakash (1983), Machac et al. (1987), Lali et al. (1989), and his own data (Chhabra, 1980). He used the following power law equation:

$$\tau = K(\gamma)^n \tag{1}$$

where τ and γ are the shear stress and shear rate respectively. The n and K are the flow behavior index and consistency index of the power law fluid. The shear rate γ is the surface-averaged particle shear rate γ_a and is defined as $(2v_t/d_p)$; where v_t and d_p are the single particle settling velocity and particle diameter respectively.

Chhabra (2006), from the dimensional analysis, has shown that the following three dimensionless groups completely characterize the settling behavior of spheres in power law fluids, in the absence of wall effects:

$$C_d = \frac{2F_d}{A_p\rho_f v_t^2} \tag{2}$$

$$Re = \frac{d_p v_t \rho_f}{\mu_f} \tag{3}$$

and n, the power law flow behavior index.

In Eqs. (2) and (3) above,

F_d	drag force
A_p	projected area
μ_f	fluid viscosity
μ_a	apparent fluid viscosity, defined as $[K\,(\gamma_a \text{ or } \gamma_m)^{n-1}]$
γ_a	average particle shear rate $(2v_t/d_p)$
γ_m	maximum particle shear rate $(3v_t/d_p)$
ρ_f	fluid density

However, dimensional considerations suggest the definition of the Reynolds number which is based on the effective viscosity to be given by $K\left(\frac{v_t}{d_p}\right)^{n-1}$. This definition has also used extensively in the literature, e.g., see Chhabra (2006).

Therefore, the drag coefficient is expected to be a function of Re and n; however, how strong the dependence of C_d is on n can only be resolved via experiments or detailed numerical simulations (Dhole et al., 2006).

From the analysis of the data of five different authors, Chhabra (1986) concluded that the drag coefficients of spheres moving in power law liquids were in line with the Newtonian standard drag curve (±30%, with maximum of 70%) without exhibiting any additional dependence on n suggested by previous researchers.

Later, Chhabra (2006) showed that the numerical simulations suggest that the C_d is a strong function of n in the low Reynolds number regime and this dependence somewhat diminishes with the increasing Reynolds number. Earlier, however, Shah (1982, 1986) had already demonstrated this fact from the analysis of his experimental data.

Therefore, further improvement in the Newtonian standard drag curve is only possible by incorporating additional dependence on the power law index, n and through the proper method suggested by Shah (1982, 1986).

3.2. Shah's approach

As noted previously, Shah (1982, 1986) recognized the additional dependence on n of the drag coefficient of particle settling velocity data in power law liquids besides the particle Reynolds number based upon the apparent viscosity. He, therefore, chose to plot the data as C_d^{2-n} versus Re instead of C_d versus Re as generally chosen by previous workers. He also used the power law fluid model as presented by Eq. (1). The only difference in Shah's analysis of the settling velocity data was that he used the definition of maximum shear rate around the particle, γ_m, as $(3v_t/d_p)$ instead of average shear rate, $(2v_t/d_p)$. Shah chose to use the maximum shear rate because this way it provides a more conservative estimate of the particle settling velocity values.

Upon rearrangement of the equation for Re (from the scaling argument), one can ascertain the order of shear rate to be (v_t/d) without ascribing any physical significance to it. On the other hand, the idea of using $(2v_t/d)$ as the representative shear rate denotes the inclusion of additional information, howsoever rudimentary or primitive. Therefore, the difference between the two definitions of the *Re* is more subtle than simply a factor of 2^{n-1}.

3.3. Re-analysis of previous authors' data

In order to re-analyze the particle settling velocity data of Dallon, Prakash, Lali et al., Chhabra, and Shah in power law liquids using Shah's approach, the Shah's data were first modified to be consistent with the other authors' data. It simply required multiplying all particle Reynolds number values of Shah by 3^{n-1}.

Table 1
Fluids considered in analysis and their rheological properties

Author	Fluid	n	K ($\mathrm{lb_f\ s^n/ft^2}$)	K ($\mathrm{Pa\ s^n}$)
Dallon	CMC	0.68–0.94	1.05×10^{-2}–1.83×10^{-3}	5.02×10^{-1}–8.7×10^{-2}
	HEC	0.64–0.77	1.3×10^{-2}–2.51×10^{-3}	6.22×10^{-1}–1.2×10^{-1}
Prakash	CMC	0.535–0.745	5.9×10^{-2}–3.6×10^{-3}	2.82–1.72×10^{-1}
Lali et al.	CMC	0.555–0.715	8.8×10^{-2}–3.76×10^{-3}	4.21–1.8×10^{-1}
Chhabra	CMC	0.76–0.89	1.36×10^{-3}–3.13×10^{-3}	6.51×10^{-2}–1.5×10^{-1}
Shah	65% Sugar solution	1.00	1.41×10^{-3}–1.96×10^{-3}	6.75×10^{-2}–9.4×10^{-2}
	HEC	0.762	9.9×10^{-4}	4.74×10^{-2}
	HPG	0.281–0.553	1.796×10^{-1}–5.74×10^{-3}	8.6–2.75×10^{-1}

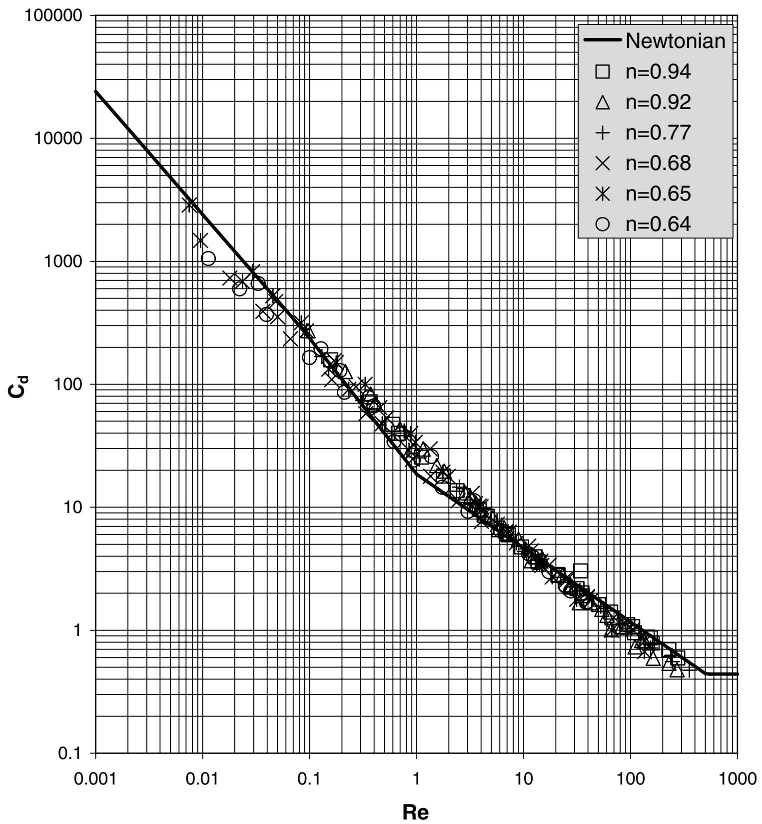

Fig. 1. Dallon's particle settling velocity data plotted as C_d versus *Re*.

In order to avoid the trial-and-error procedure, Shah suggested plotting the data on a logarithmic paper as $\sqrt{C_d^{2-n}Re^2}$ versus Re. Therefore, the data set of each author was analyzed in this manner. The resultant curve of the data was least-squares curve fitted using the following form of the equation:

$$\sqrt{C_d^{2-n}Re^2} = A(Re)^B \tag{4}$$

where A and B are constants.

First, the data of each author were analyzed separately and correlations were developed for each case. Later, all data sets were combined and a new model was developed from the entire data set.

For comparison with the Newtonian standard drag curve, the following standard equations from the literature (McCabe and Smith, 1956) for the creeping or Stokes region, intermediate region, and turbulent region were used. For continuity, both the Stokes and intermediate region equations were extended to unity Reynolds number.

Stokes region:

$$C_d = 24/Re \quad (Re < 1.0) \tag{5}$$

Intermediate region:

$$C_d = 18.5/(Re^{0.6}) \quad (1.0 < Re < 500) \tag{6}$$

Turbulent region:

$$C_d = 0.44 \quad (500 < Re < 200,000) \tag{7}$$

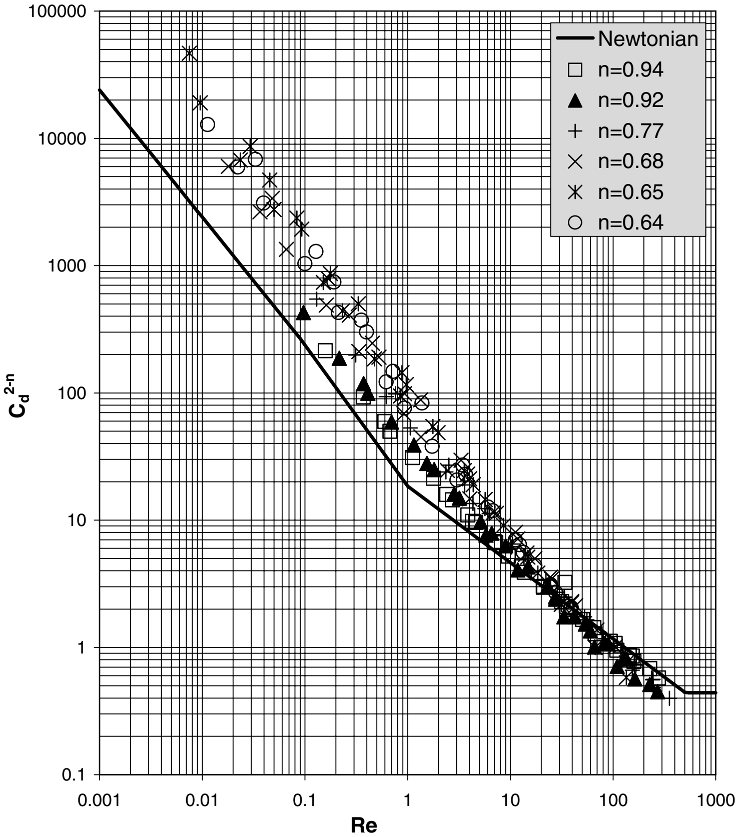

Fig. 2. Dallon's particle settling velocity data plotted as C_d^{2-n} versus Re.

Table 1 provides a summary of the types of fluids and the corresponding values of the power law constants, n and K for various data sets used in this work. More details can be found elsewhere (El Fadili, 2005).

4. Results and discussion

Fig. 1 depicts Dallon's particle settling velocity data in various power law fluids plotted on a logarithmic scale as C_d versus Re. The Newtonian standard drag curve, approximated here by Eqs. (5)–(7), is also shown for comparison. It can be seen that all data are scattered around the Newtonian curve and no clear trend is evident. Same data are re-plotted as C_d^{2-n} versus Re on a logarithmic scale in Fig. 2. The data now seem to show a family of individual curves as a function of the power law flow behavior index, n. The data behavior is no longer obscured as seen in Fig. 1. This plot not only confirms the validity of this approach but also clearly indicates the additional dependence of drag coefficient on the power law flow behavior index, n.

Similar plots for the data of Prakash, Lali et al., Chhabra, and Shah are shown in Figs. 3–8. It is evident from all these plots that a well-behaved data are seen when plotted using the Shah's approach than the modified Newtonian standard drag curve.

4.1. Results of individual data set

A total of 391 data points were used featuring 21 values of the power law flow behavior index, n, that ranged from 0.281 to 1.0. Table 2 shows the absolute percent deviation [absolute % deviation = $\{$(experimental C_d − model C_d)/experimental $C_d\} \times 100$] between the experimental data of the drag coefficient and the model predictions based on the Shah approach and the absolute percent deviation [absolute % deviation = $\{$(Experimental C_d − Newtonian C_d)/experimental $C_d\} \times 100$] between the experimental data and Newtonian standard

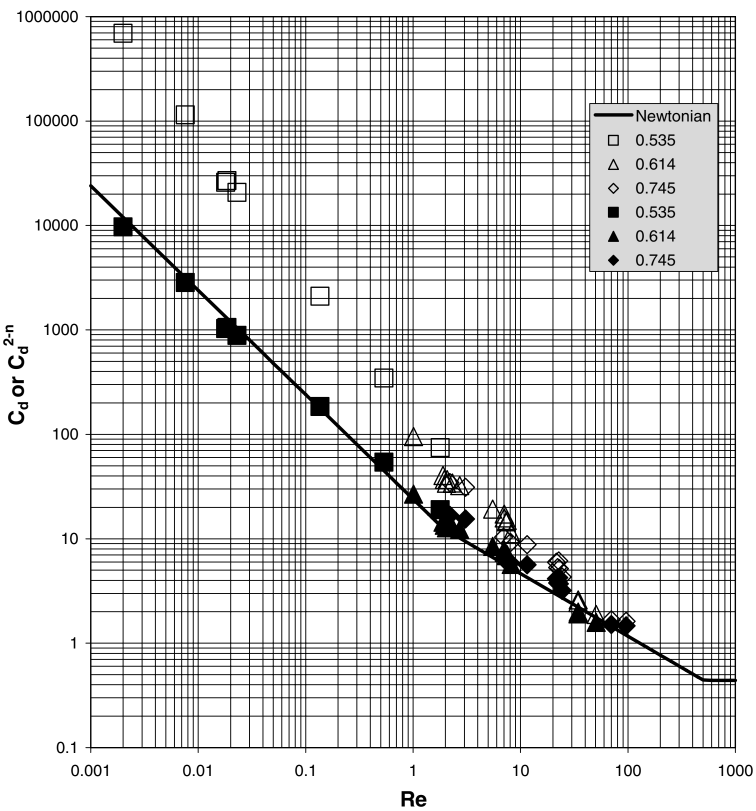

Fig. 3. Prakash's particle settling velocity data plotted as C_d and C_d^{2-n} versus Re (solid symbols for C_d and open symbols for C_d^{2-n}).

drag curve. Almost in all cases, the new model correlates the experimental data significantly better than the Newtonian model. The results summarized in Table 2 demonstrate that the minimum and maximum percent deviations between the experimental data and model are 4.07 and 15.09 while between the experimental data and Newtonian drag curve they are 10.16 and 37.83.

Statistical analysis of the experimental data with Newtonian curve predictions revealed that out of 391 data points, 127 data points showed a deviation between 10% and 20%, 62 points showed a deviation between 20% and 30% and 74 data points had deviation of more than 30%, with a maximum deviation of 118%. Only 128 points had deviation less than 10%.

A similar comparison of the experimental data with the model prediction revealed that 234 points showed a deviation of less than 10% deviation, 112 data points showed deviation between 10% and 20%, and 34 points had deviation in the range 20–30%. Only 11 points deviated more than 30% with a maximum deviation of 53%.

The above statistical analysis clearly suggests the improvement in the predictions of drag coefficient (particle settling velocity) using the proposed model over the modified Newtonian standard drag curve as suggested by some researchers.

4.2. Development of new model

It is clearly shown that the modified Newtonian standard drag curve is not best suited to describe the experimental data of particle settling in non-Newtonian power law fluids. The need for a simple and reliable model that can be applied to the prediction of drag coefficient for single particle moving in both Newtonian and

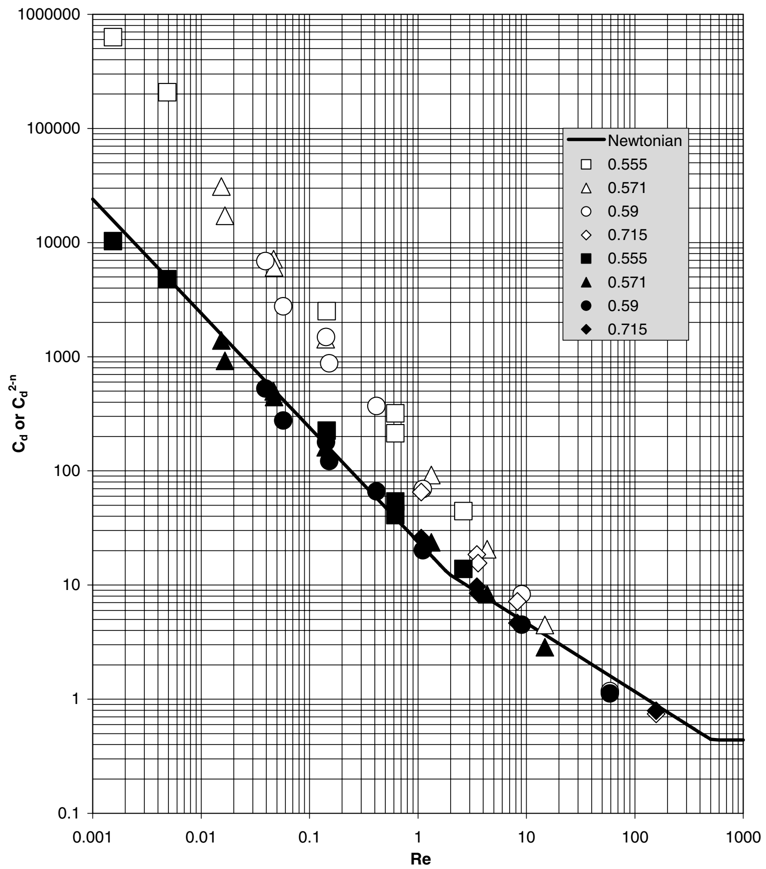

Fig. 4. Lali's particle settling velocity data plotted as C_d and C_d^{2-n} versus Re (solid symbols for C_d and open symbols for C_d^{2-n}).

power law non-Newtonian fluids is thus obvious. For this purpose, the parameters, A and B of Eq. (4) were plotted as a function of the power law fluid index, n. Figs. 9 and 10 depict the data of A and B as a function of n and also show the least-squares curve fit of the data. The data fit show R^2 values of 0.92 and 0.89 for the parameters A and B respectively. The following equations are obtained:

$$A = 6.9148(n^2) - 24.838(n) + 22.642 \tag{8}$$

$$B = -0.5067(n^2) + 1.3234(n) - 0.1744 \tag{9}$$

Considering data of five different investigations and the number of data points used, the trends obtained in Figs. 9 and 10 can be considered very well. It is always possible to improve the degree of fit by introducing additional fitted parameters. In view of the experimental uncertainty, we believe the degree of fit to be adequate. Thus, the proposed new model is as follows:

$$\sqrt{C_d^{2-n} Re^2} = A(Re)^B \tag{10}$$

where the apparent effective viscosity μ_a is defined as $K(2v_t/d_p)^{n-1}$.

Table 3 presents the summary of all absolute average percent deviations as a result of the comparison of experimental data with the new model and Newtonian model predictions. An inspection of the data shown in Table 3 reveals that as earlier, the new model is as good as or better than the Newtonian model in correlating the particle settling data of power law liquids. However, the improvement using the new model is not as good as the individual model, but nevertheless still better than the Newtonian model. The new method performed better in 13 cases out of 21. Aside from this, the variability of the results from one study to another is also an issue. This difficulty is further accentuated due to the uncertainty of wall effects, possible visco-elastic

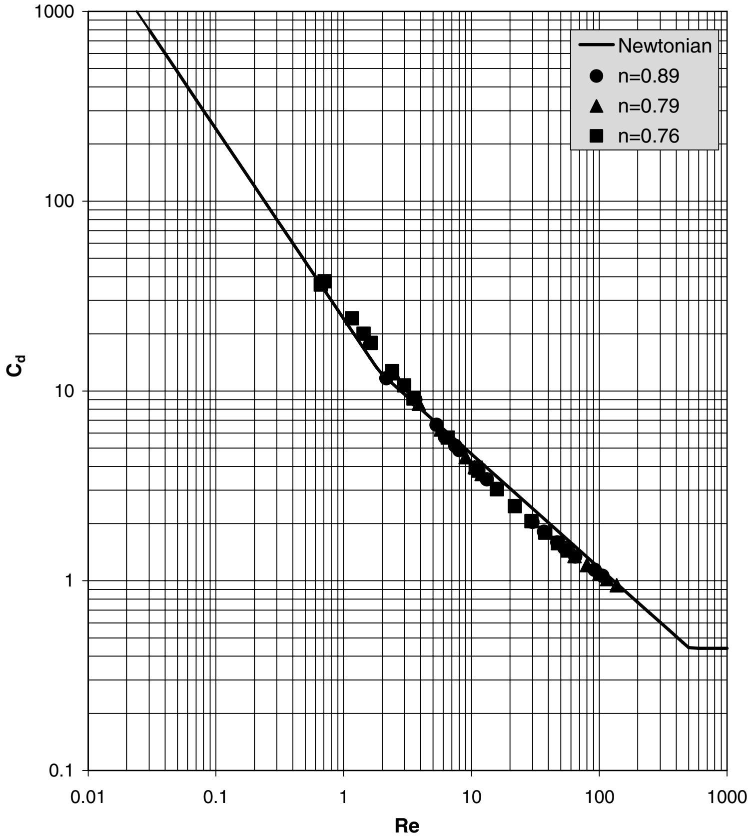

Fig. 5. Chhabra's particle settling velocity data plotted as C_d versus Re.

effects. So, wherever adequate information was reported, the experimental data were screened, but in the absence of such details, one has to take the results on their face value, which adds to uncertainty.

The statistical analysis of the experimental and new model predictions showed that out of 391 points, 164 points had a deviation of less than 10%, 110 points were within 10% and 20%, 73 points within 20% and 30%, and only 44 points had deviation in excess of 30% with a maximum deviation of 60%. These deviations are still much improved compared to the similar values obtained in a comparison between the experimental data and the predictions from the Newtonian model.

It should be noted that the new model presented by Eq. (10) is valid for the power law index, n, values in the range of 0.281–1.0 and the particle Reynolds number values in the range of 0.001–1000.

It is obvious that for maximum accuracy, an individual model of the least-squares curve fit of a given data set will be the best model to use. However, in absence of the particle settling velocity data for a given power law fluid, it is recommended that the new model be used to predict the settling velocity of the chosen particle in a given power law fluid.

4.3. Example calculation for the determination of settling velocity

4.3.1. Problem

It is desired to estimate the terminal particle settling velocity of the 20/40 mesh sand particle in carboxymethyl cellulose (CMC) fluid. The fluids rheological properties are: $n = 0.76$, $K = 6.51 \times 10^{-2}\,\mathrm{Pa\ s}^n$ and density $\rho_f = 1.0011 \times 10^3\ \mathrm{kg/m^3}$. The particle properties are: average particle diameter $d_p = 6.4 \times 10^{-4}$ m and density $\rho_p = 2.65 \times 10^3\ \mathrm{kg/m^3}$.

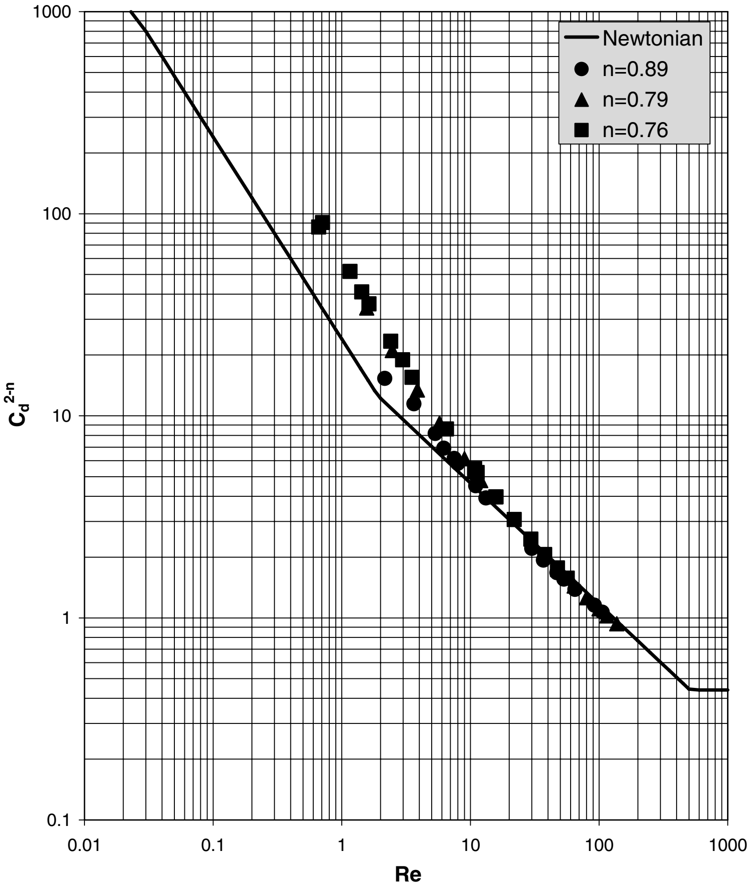

Fig. 6. Chhabra's particle settling velocity data plotted as C_d^{2-n} versus Re.

4.3.2. Solution

1. First of all, knowing n one can calculate the constants A and B using Eqs. (8) and (9) as follows:

$$A = 6.9148(0.76)^2 - 24.838(0.76) + 22.642 \quad \text{or} \quad A = 7.76$$

and

$$B = -0.5067(0.76)^2 + 1.3234(0.76) - 0.1744 \quad \text{or} \quad B = 0.54.$$

2. The non-dimensional term $\sqrt{C_d^{2-n} Re^2}$ is then calculated as follows:
By definition C_d and Re for the power law fluid (based upon the average shear rate, γ_a) are given as

$$C_d = \frac{4}{3}\left(\frac{d_p g}{v_t^2}\right)\left(\frac{\rho_p - \rho_f}{\rho_f}\right)$$

and

$$Re = \left(\frac{d_p^n v_t^{2-n} \rho_f}{(2)^{n-1} K}\right)$$

In SI units, C_d^{2-n} and Re^2 can be written as

$$C_d^{2-n} = (13.08)^{(2-n)}\left(\frac{d_p^{2-n}}{v_t^{2(2-n)}}\right)\left(\frac{\rho_p - \rho_f}{\rho_f}\right)^{2-n}$$

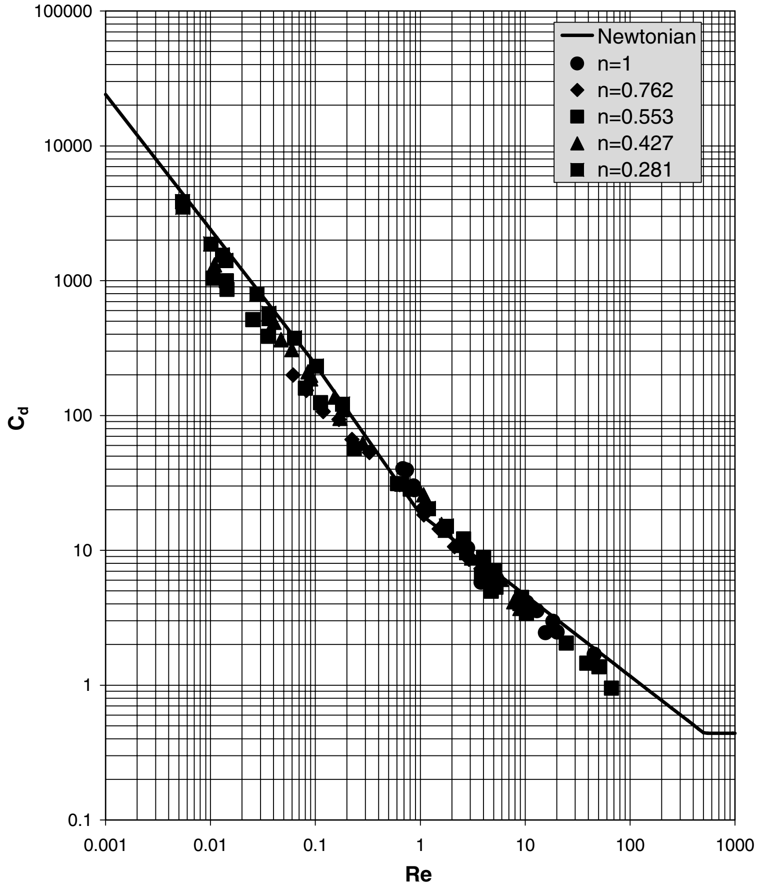

Fig. 7. Shah's particle settling velocity data plotted as C_d versus Re.

and

$$Re^2 = \left(\frac{d_p^{2n} v_t^{2(2-n)} \rho_f^2}{(2)^{2(n-1)} K^2}\right)$$

Therefore,

$$\sqrt{C_d^{2-n} Re^2} = \sqrt{\left[\frac{(13.08)^{2-n}}{(2)^{2(n-1)}}\right]\left[\frac{d_p^{n+2} \rho_f^n (\rho_p - \rho_f)^{2-n}}{K^2}\right]}$$

where

d_p particle diameter (m)
K consistency index (Pa s^n)
n fluid flow behavior index, dimensionless
v_t particle terminal velocity (m/s)
ρ_f fluid density (kg/m^3)
ρ_p particle density (kg/m^3)

Hence,

$$\sqrt{C_d^{2-n} Re^2} = \sqrt{\left[\frac{(13.08)^{2-0.76}}{(2)^{2(0.76-1)}}\right]\left[\frac{(0.00064)^{0.76+2} \times 1001.1^{0.76}(2650 - 1001.1)^{2-0.76}}{(6.51 \times 10^{-2})^2}\right]}$$

or $\sqrt{C_d^{2-n} Re^2} = 4.77$.

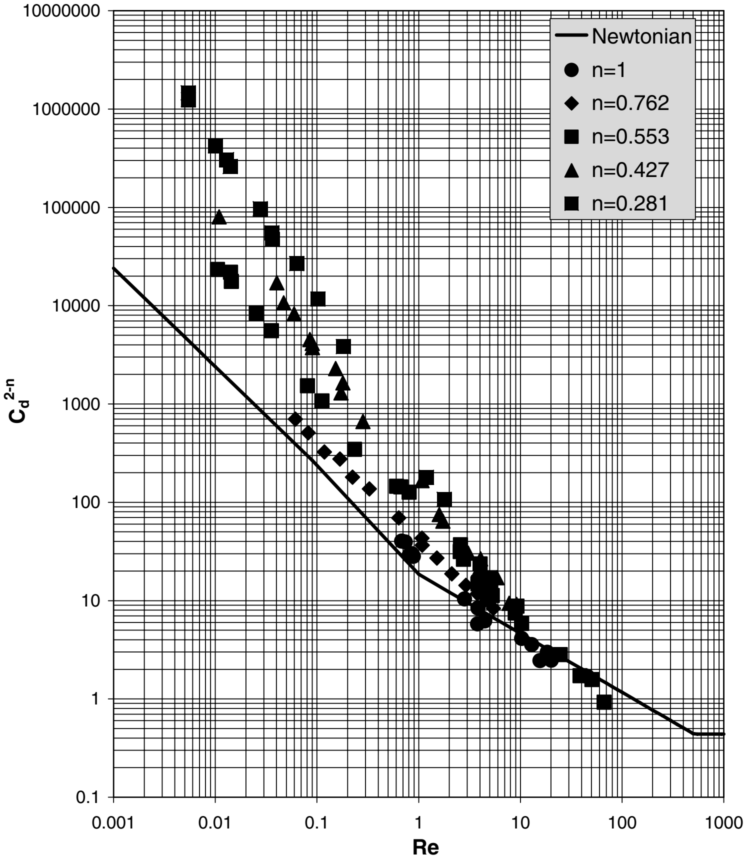

Fig. 8. Shah's particle settling velocity data plotted as C_d^{2-n} versus Re.

Table 2
Deviations of individual model and Newtonian model predictions from experimental data

Author	*n*	*A*	*B*	R^2	% Dev. in Cd Model – Exptl	% Dev. in Cd Newt – Exptl
Shah	1.00	5.01	0.61	0.99	12.85	14.31
	0.762	6.53	0.50	0.99	4.30	37.83
	0.553	9.91	0.41	0.98	13.14	36.88
	0.427	12.55	0.32	0.99	7.75	26.15
	0.281	15.98	0.18	0.97	4.78	15.69
Chhabra	0.89	5.04	0.65	0.99	4.07	10.16
	0.79	6.24	0.61	0.99	7.53	10.39
	0.76	7.44	0.54	0.99	6.73	14.62
Dallon	0.94	5.88	0.62	0.99	11.79	10.87
	0.92	6.61	0.57	0.99	9.98	21.99
	0.68	9.58	0.49	0.99	12.21	18.41
	0.65	10.54	0.43	0.99	11.38	20.90
	0.64	9.92	0.45	0.99	10.95	21.70
	0.77	7.79	0.56	0.99	9.14	12.86
Prakash	0.535	12.14	0.33	0.99	7.08	19.80
	0.614	9.12	0.53	0.98	9.97	16.05
	0.745	8.06	0.59	0.98	9.76	21.95
Lali	0.555	12.47	0.34	0.98	12.45	23.00
	0.571	11.46	0.37	0.99	6.58	20.52
	0.59	10.79	0.41	0.98	15.09	22.77
	0.715	7.43	0.56	0.99	10.54	13.37

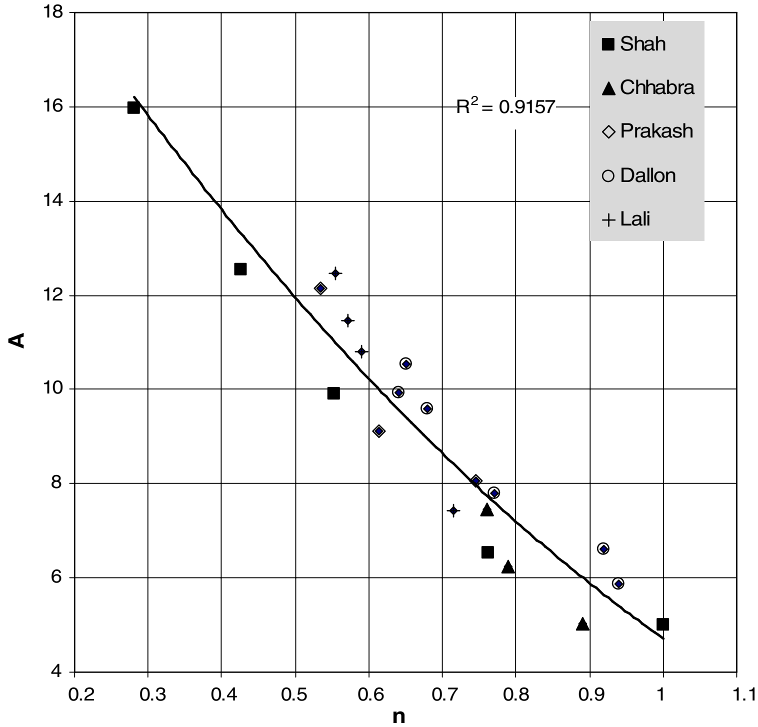

Fig. 9. Parameter *A* versus power law flow behavior index, *n*.

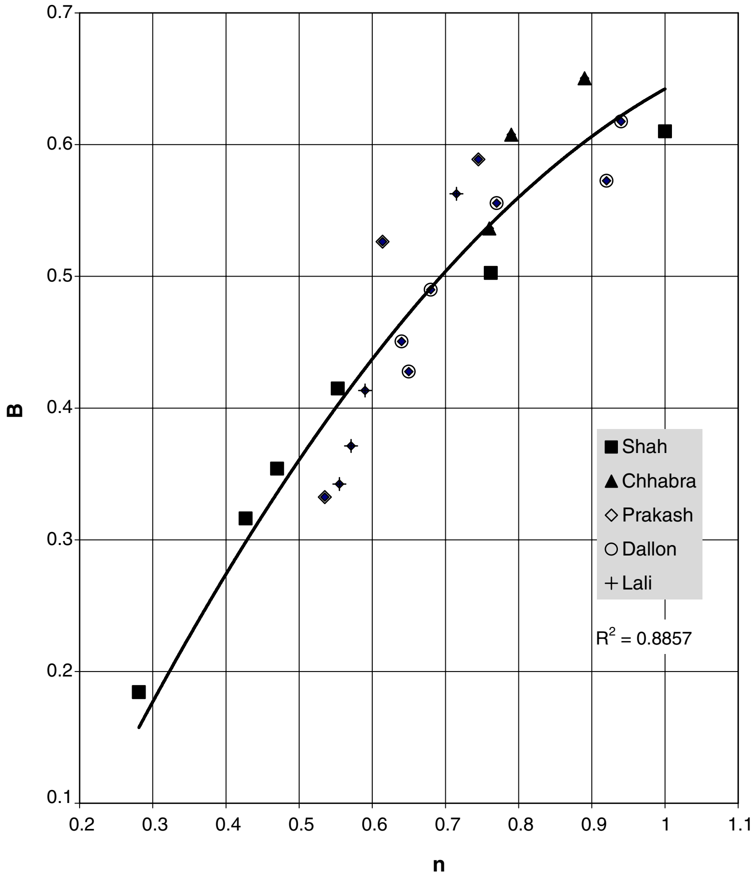

Fig. 10. Parameter B versus power law flow behavior index, n.

3. Calculate the particle Reynolds number from the new model as follows:

$$\sqrt{C_{\mathrm{d}}^{2-n}Re^2} = A(Re)^B$$

or

$$Re = \left(\frac{4.77}{7.76}\right)^{1/0.54}$$

Therefore, $Re = 0.41$.

4. Finally, knowing Re from step 3 above, the particle settling velocity is calculated as follows:

$$v_{\mathrm{t}} = \left[\frac{(2)^{n-1}KRe}{d_{\mathrm{p}}^{n}\rho_{\mathrm{f}}}\right]^{\frac{1}{(2-n)}} = \left[\frac{(2)^{0.76-1}\times 0.0651\times 0.41}{0.00064^{0.76}\times 1.0011\times 10^{3}}\right]^{\frac{1}{(2-0.76)}}$$

Therefore,

$$v_{\mathrm{t}} = \underline{1.62\times 10^{-2}}\ \mathrm{m/s}.$$

Hence, the 20/40 mesh sand particle will settle in a given CMC fluid with a terminal settling velocity of $\underline{1.62\times 10}^{-2}$ m/s or $\underline{0.972}$ m/min.

Table 3
Deviations of new model and Newtonian model predictions from experimental data

Author	n	A	B	% Dev. in Cd Model – Exptl	% Dev. in Cd Newt – Exptl
Shah	1	4.72	0.64	15.09	14.31
	0.762	7.73	0.54	28.10	37.83
	0.553	11.02	0.40	19.71	36.88
	0.427	13.30	0.30	12.70	26.15
	0.281	16.21	0.16	12.88	15.69
Chhabra	0.76	7.76	0.54	9.74	14.62
	0.79	7.34	0.55	11.83	10.39
	0.89	6.01	0.60	12.58	10.16
Dallon	0.94	5.40	0.62	12.41	10.87
	0.92	5.64	0.61	13.93	21.99
	0.68	8.95	0.49	14.94	18.41
	0.65	9.42	0.47	16.08	20.90
	0.64	9.58	0.47	10.66	21.70
	0.77	7.62	0.54	10.88	12.86
Prakash	0.535	11.33	0.39	27.14	19.80
	0.571	10.71	0.42	17.54	20.52
	0.59	10.39	0.43	13.86	22.77
Lali	0.715	8.42	0.51	15.89	13.37
	0.555	10.99	0.40	28.52	23.00
	0.614	10.00	0.45	15.80	16.05
	0.745	7.98	0.53	22.85	21.95

5. Conclusions

A new model is presented to estimate single spherical particle settling velocity in a power law shear-thinning liquid. The model is valid for the power law flow behavior index, n in the range of 0.281–1.00 and the particle Reynolds number range of 0.001–1000. The model is developed with the 391 experimental data points available in the literature from five different researchers. The data of three fluid types, CMC, HEC, and HPG and each with different concentrations have been used. The new model proposed in this study, in the limited case, reduces to a Newtonian fluid.

The new model is found to be an improvement to the existing models in the literature to predict the spherical particle settling velocity in power law liquids. The individual model for each data set obviously yields much better results than the new model. However, the new model is recommended in absence of the experimental data set and when there is a need for the estimation of settling velocity of spherical particle in power law liquid.

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